§12-2 Geometric Sequences

- A geometric sequence is…
  - A sequence of numbers in which the ratio between terms is constant.
    - The ratio between terms is called the ***common ratio***, *r*.
  - A sequence in which you obtain the next term by multiplying the same number to the previous term each time.
- Examples:
  - 4, 8, 16, 32, 64, 128, …
  - $9, -3, 1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$

§12-2 Geometric Sequences

- Determine if the sequence is geometric. If it is, state the common ratio.
  - $16, 4, 1, \frac{1}{4}, \ldots$

$\frac{4}{16}$ $\frac{1}{4}$ $\frac{\frac{1}{4}}{1}$

Yes, it's geometric; $r = \frac{1}{4}$.

§12-2 Geometric Sequences

- Determine if the sequence is geometric. If it is, state the common ratio.
  - $5, 30, 180, 1080, \ldots$

$\frac{30}{5}$ $\frac{180}{30}$ $\frac{1080}{180}$

Yes, it's geometric; $r = 6$.

§12-2 Geometric Sequences

- Determine if the sequence is geometric. If it is, state the common ratio.
  - $7, 14, 21, 28, \ldots$

$\frac{14}{7}$ $\frac{21}{14}$

No, it's not geometric.

§12-2 Geometric Sequences

- Just like arithmetic sequences, once you know a little information about a sequence, you can determine a formula for a sequence.
- Then, you can use that formula to find any term of the sequence you're interested in.
- Example:
  - Use the sequence: 9, 18, 36, 72, 144, …
  - What is the 10th term of the sequence?
    - We don't want to have to list out all the terms do we?
    - So let's find a shorter way by doing a bit of thinking.

§12-2 Geometric Sequences

- Example:
  - Use the sequence: 9, 18, 36, 72, 144, …
  - What is the 10th term of the sequence?
    - Every time we get a new term, how much are we multiplying by? (What is the common ratio?)
      - 2
    - So, to get the 3rd term, how many 2s did we multiply by?
      - 9, 18, 36….

        2 2
      - To get the **<u>3</u>**rd term we multiplied by **<u>2</u>** 2s.
      - So how many 2s do you think we will need to add to get the 10th term?
        - 9 is correct!

§12-2 Geometric Sequences

- Example:
  - Use the sequence: 9, 18, 36, 72, 144, …
  - What is the 10th term of the sequence?
    - How many 2s do you think we will need to add to get the 10th term?
      - 9 is correct!
    - What number did we start with? (What is a_1?)
      - $a_1 = 9$
    - So, we start with 9, then we multiply by 9 2s. How much is 9 2s?
      - $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$
      - $2^9 = 512$
    - Therefore, $a_{10} = 9 \cdot 512 \quad = 4{,}608$.

§12-2 Geometric Sequences

- Let's formalize that by taking out the numbers and replacing them all with variables.
  - What we did was we said to find any number term (the *n*th term), a_n
    - You find 1 less than that term, $n-1$
    - Multiply the common ratio, *r*, by itself that many times
    - Then multiply it to the value of the first term, a_1
  - Therefore, the formula for any geometric sequence is: $a_n = a_1 \cdot r^{(n-1)}$

§12-2 Geometric Sequences

- Let's try it again.
- Find the 102nd term in the sequence: -50, 50, -50, 50, -50, …
  - $r =$ -1
  - $a_1 =$ **-50**
  - $a_n = a_1 \cdot r^{n-1}$
  - $a_{102} \quad \cdot (\quad)^{102-1}$
  - $a_{102} = -50 \cdot (-1)^{101}$
  - $a_{102} = -50 \cdot (-1)$
  - $a_{102} = 50$